A Table of Digital Frequency Notation

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When we read the literature of digital signal processing (DSP) we encounter a number of different, and equally valid, ways to algebraically represent the notion of frequency for discrete-time signals. (By **frequency** I mean a measure of *angular repetitions per unit of time*.)

The various mathematical expressions for sinusoidal signals use a number of different forms of a frequency variable and the units of measure (dimensions) of those variables are different. It's sometimes a nuisance to keep track of those different algebraic frequency variables. Add to this the fact that the time-index variable *n* is sometimes dimensionless, and sometimes *n* is measured in samples.

The following table presents a list of algebraic expressions that I have seen in the literature of DSP. I keep a copy of that table pinned to the wall next to my desk.

For simplicity I show no initial phase term in the sinusoidal algebraic expressions in **bold** font in the left column of the table. For reference, I've included two sinusoidal expressions for continuous-time (analog) sine waves at the top of the table.

Table resides on next page.

Notation	Frequency variable [frequency range]	Units (Dimensions)
$\sin(2\pi f_o t)$ [Analog]	f_o in cycles/second (Hz) $[-F_s/2 \le f_o \le F_s/2]$	$f_o t$ is $\frac{\text{cycles}}{\text{second}} \cdot \text{seconds}$ = cycles.
$\sin(\Omega_o t)$ [Analog]	Ω_o in radians/second $[-\pi F_s \le \Omega_o \le \pi F_s]$	$\Omega_o = 2\pi f_o$. f_o in cycles/second. $\Omega_o t$ is $\frac{\text{radians}}{\text{second}} \cdot \text{seconds}$ = radians.
$\sin(2\pi f_o n t_s)$ [Digital]	f_o in cycles/second $[-F_s/2 \le f_o \le F_s/2]$	n in samples. $f_o n t_s$ is $\frac{\text{cycles}}{\text{second}} \cdot \text{samples} \cdot \frac{\text{seconds}}{\text{sample}}$ = cycles.
$\sin(2\pi n f_o/f_s)$ [Digital]	f_o/f_s in cycles/sample $[-1/2 \le f_o/f_s \le 1/2]$	n in samples. $n f_o/f_s$ is $\text{samples} \cdot \frac{\text{cycles}}{\text{second}} \cdot \frac{\text{seconds}}{\text{sample}}$ = cycles.
$\sin(2\pi f_o n)$ [Digital]	f_o in cycles/sample $[-1/2 \le f_o \le 1/2]$	n in samples. $f_o n$ is $\frac{\text{cycles}}{\text{sample}} \cdot \text{samples}$ = cycles.
$\sin(\omega_o n)$ (See row below) [Digital]	ω_o in radians/sample $[-\pi \le \omega_o \le \pi]$	n in samples. $\omega_o n$ is $\frac{\text{radians}}{\text{sample}} \cdot \text{samples}$ = radians.
$\sin(\omega_o n)$ [Digital]	ω_o in radians $[-\pi \le \omega_o \le \pi]$	n is dimensionless. $\omega_o n$ is = radians.

t = continuous time in seconds

f_s = sample rate in samples/second

$t_s = 1/f_s$ in seconds/sample

F_s = sample rate in cycles/second (Hz)